

# Continuous-Variable Quantum Information Distributor: Reversible Telecloning

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We propose a scheme of continuous-variable reversible telecloning, which broadcast the information of an unknown state without loss from a sender to several spatially separated receivers exploiting multipartite entanglement as quantum channels. In this scheme, quantum information of an unknown state is distributed into  $M$  optimal clones and  $M - 1$  anticlones using  $2M$ -partite entanglement. For the perfect quantum information distribution that is optimal cloning,  $2M$ -partite entanglement is required to be a maximum two-party entanglement. Comparing with the quantum telecloning proposed by Loock and Braunstein [Phys. Rev. Lett. **87**, 247901 (2001)], this protocol produces the anticlones (or time-reversed state) of the unknown quantum state, thus, keep all information of an unknown state.

*Introduction.* — One of the main tasks in quantum information processing and quantum computation is the distribution of quantum information encoded in the states of quantum systems. The perfect distribution requires the no loss of the quantum information of the unknown state, that means this process is reversible and the unknown state can be reconstructed in a quantum system again. It is now well known that quantum information can not be exactly copied[1]. Although exact cloning is impossible, one can construct approximate cloning machines. Buzek and Hillery proposed a universal quantum cloning machine for an arbitrary quantum state where the copying process is independent of the input states[2]. In recent years, quantum information and communication have been extended to the domain of continuous variable (CV)[3], due to relative simplicity and high efficiency in the generation, manipulation, and detection of CV state. To date, the CV local cloning has been studied intensively[4, 5, 6, 7, 8, 9].

In parallel, quantum nonlocal cloning (telecloning) has also been intensively studied, which is a combination of quantum cloning and teleportation performed simultaneously. The aim of telecloning is to broadcast information of an unknown state from a sender to several spatially separated receivers exploiting multipartite entanglement as quantum channels. For qubits, Bruß et al. first proposed  $1 \rightarrow 2$  telecloning, which use nonmaximum tripartite entanglement (here it is named as irreversible teleclone states)[10]. In this case, the anticlones (phase-conjugate clones, or time-reversed state) are lost, thus, quantum channel don't require maximum entanglement. This kind of telecloning is called irreversible telecloner and is regarded as imperfect nonlocal distributor of quantum information. More generally,  $1 \rightarrow M$  irreversible teleclone states, which are  $M + 1$ -partite entanglement, are given in Ref.[11]. Later, Murao et al. proposed a new  $1 \rightarrow M + (M - 1)$  telecloning scheme, in which quantum information of an input qubit is distributed into  $M$  optimal clones and  $M - 1$  anticlones using  $2M$ -partite entanglement[12]. This kind of telecloning is called reversible telecloner and is regarded as perfect nonlocal

distributor of quantum information. Due to no loss of quantum information,  $2M$ -partite entanglement is required to be a maximum two-partite entanglement. More generally, qubit telecloning with  $N$  identical inputs distributed among  $M$  receivers has been studied[13]. For continuous variables, Loock and Braunstein proposed optimal  $1 \rightarrow M$  telecloning of coherent states via a  $M + 1$ -partite entangled state[14]. It is emphasized in the protocol that optimal telecloning can be achieved by exploiting nonmaximum bipartite entanglement between the sender and all receivers. This result is not surprising since the anticlones are not produced in this protocol and the quantum information of the unknown state is lost in the process of distribution. This scheme is regarded as the CV irreversible telecloner and corresponds to the irreversible telecloner in the domain of discrete variables[10, 11]. Furthermore, the CV irreversible telecloning was studied in noise environment[15]. Recently, irreversible telecloning of optical coherent states was demonstrated experimentally[16]. In this Letter, we propose a scheme of CV reversible telecloning, which broadcast the information of an unknown state without loss from a sender to several spatially separated receivers exploiting multipartite entanglement as quantum channels. In this process, quantum information of an unknown state is distributed into  $M$  optimal clones and  $M - 1$  anticlones using  $2M$ -partite entanglement. For the reversible telecloning (the perfect quantum information distribution) that is optimal cloning,  $2M$ -partite entanglement used for quantum channel is required to be a maximum two-party entanglement. Further, we generalize  $1 \rightarrow M + (M - 1)$  quantum telecloning to  $N \rightarrow M + (M - N)$  case and also provide an explicit design of an asymmetric reversible telecloning. Like the quantum teleportation, we give the lower and upper bounds to achieve quantum telecloning when the imperfect quantum entanglement is utilized.

*$1 \rightarrow 2 + 1$  telecloning.* — A schematic setup for CV  $1 \rightarrow 2 + 1$  telecloning is depicted in Fig.1. The quantum states we consider in this Letter can be described using the electromagnetic field annihilation oper-



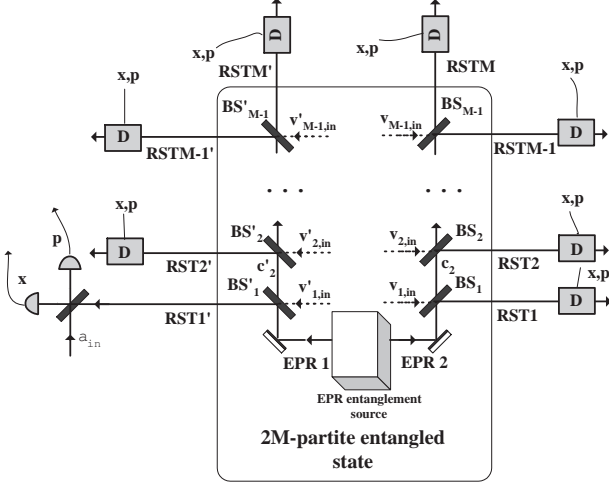


FIG. 2: A schematic diagram of  $1 \rightarrow M + (M-1)$  telecloning.

Dan perform the joint (Bell) measurement respectively on their entangled modes and clones (anticlone). Receiving these measurement results from Bob, Clair and Dan, Alice displaces her entangled mode and can generate the original unknown state. However, the unknown state can not be reconstructed only with two optimal clones. It is worth noting that the optimal two clones and anticlone in Bob, Clair and Dan constitute a tripartite entangled state, which exactly corresponds to the  $1 \rightarrow 2$  CV irreversible telecloning state[14].

In real experiment, a maximally EPR entangled state is not available because of finite squeezing and inevitable losses. To assess the quality of telecloning like teleportation, we apply the fidelity measure  $\mathcal{F} \equiv \langle \psi^{in} | \hat{\rho}^{out} | \psi^{in} \rangle$ [18]. In the case of unity gain, the fidelity for the Gaussian states is simply given by  $\mathcal{F} = 2/\sqrt{(1 + \langle \delta \hat{X}_{out}^2 \rangle)(1 + \langle \delta \hat{Y}_{out}^2 \rangle)}$ . For the classical case of  $r = 0$ , i.e., the EPR beams were replaced by uncorrelated vacuum inputs, the fidelity of Bob and Clair's outputs is found to be  $\mathcal{F}_{class} = 1/2$ [19], which corresponds to the classical limit for coherent state cloning. The fidelity of Dan's anticlone is  $\mathcal{F}_{antic} = 1/2$ , which is independent on the quantum entanglement. When they share quantum entanglement  $r > 0$ , the fidelity of the clones of Bob and Clair is  $\mathcal{F}_{clone} = 2/(3 + e^{-2r})$ . It is clearly shows that Bob and Clair get the clones with fidelity  $\mathcal{F}_{clone} > 1/2$ , thus the quantum  $1 \rightarrow 2 + 1$  telecloning of coherent states is deemed successful. Note that the optimal fidelity of  $1 \rightarrow 2 + 1$  coherent-state reversible telecloning is  $2/3$  for the clones and  $1/2$  for the anticlone, which requires the maximally EPR entangled state.

**$1 \rightarrow M + (M-1)$  telecloning.** — We now generalize  $1 \rightarrow 2 + 1$  quantum telecloning to  $1 \rightarrow M + (M-1)$ , which produces  $M$  clones and  $M-1$  anticlones from a single in-

put state using  $2M$ -partite entanglement. We first generate the  $2M$ -partite entanglement by a sequence of a EPR entangled beams and  $2(M-1)$  beam splitters with appropriately adjusted transmittances and reflectances as illustrated in Fig.2. The modes  $\hat{v}_{j,in}$  and  $\hat{v}'_{j,in}$  are in the vacuum state. The EPR entangled modes  $\hat{a}_{EPR1}$  and  $\hat{a}_{EPR2}$  are mixed with  $\hat{v}_{1,in}$  and  $\hat{v}'_{1,in}$  at the beam splitters  $BS'_1$  and  $BS_1$ , respectively. The mode  $\hat{a}_{RTS1'}$  ( $\hat{a}_{RTS1}$ ) contains the EPR entangled mode  $\hat{a}_{EPR1}$  ( $\hat{a}_{EPR2}$ ) by a factor of  $1/\sqrt{M}$ . The output  $\hat{c}'_2$  ( $\hat{c}_2$ ) is split at the  $BS'_2$  ( $BS_2$ ) and so on, until we reach the last beam splitter  $BS'_{M-1}$  ( $BS_{M-1}$ ). The transformation performed by the  $j$ th beam splitter can be written as

$$\begin{aligned} \hat{a}_{RTSj^{(v)}} &= \sqrt{\frac{1}{M-j+1}} \hat{c}_j^{(v)} + \sqrt{\frac{M-j}{M-j+1}} \hat{v}_{j,in}^{(v)}, \\ \hat{c}_{j+1}^{(v)} &= \sqrt{\frac{M-j}{M-j+1}} \hat{c}_j^{(v)} - \sqrt{\frac{1}{M-j+1}} \hat{v}_{j,in}^{(v)}, \end{aligned} \quad (3)$$

where  $\hat{c}'_1 = \hat{a}_{EPR1}$ ,  $\hat{c}_1 = \hat{a}_{EPR2}$ , and  $\hat{a}_{RTSM^{(v)}} = \hat{c}_M^{(v)}$ . It is clearly shows that each  $2M$ -partite entangled mode  $\hat{a}_{RTSj^{(v)}}$  (or  $\hat{a}_{RTSj}$ ) contains  $1/M$  portion of the the EPR entangled mode  $\hat{a}_{EPR1}$  (or  $\hat{a}_{EPR2}$ ) and  $(M-1)/M$  portion of the vacuum noise. The entanglement structure of  $2M$ -partite telecloning state is also divided two sets  $\{\hat{a}_{RTS1'}, \hat{a}_{RTS2'}, \dots, \hat{a}_{RTSM'}\}$  and  $\{\hat{a}_{RTS1}, \hat{a}_{RTS2}, \dots, \hat{a}_{RTSM}\}$ . The parties in the same set have no any quantum entanglement, however, any two parties lying different sets respectively have bipartitely entanglement.

For quantum  $1 \rightarrow M + (M-1)$  telecloning, the sender chooses any one of  $2M$  modes of the telecloning state and performs a joint measurement on his entangled mode and an unknown input mode  $\hat{a}_{in}$ . Then the sender informs the other parties of its measurement results  $x$  and  $p$ . After receiving these measurement results from sender, each party displaces its entangled mode by modulating the amplitude and phase of an auxiliary beam, then combining  $1/99$  beam splitter. The parties in the different set with the sender produce the clones with  $-g_x = g_p = \sqrt{2}$  and the parties in the same set with the sender produce the anticlones with  $g_x = g_p = -\sqrt{2}$ . The fidelity of  $M$  clones and  $M-1$  anticlones is given by

$$\begin{aligned} \mathcal{F}_{clone}^{1 \rightarrow M+(M-1)} &= \frac{M}{2M-1 + e^{-2r}}, \\ \mathcal{F}_{antic}^{1 \rightarrow M+(M-1)} &= \frac{1}{2}. \end{aligned} \quad (4)$$

The classical limit for  $1 \rightarrow M + (M-1)$  quantum telecloning is  $\mathcal{F}_{class} = 1/2$ . The fidelity of the anticlones is  $\mathcal{F}_{antic} = 1/2$ , which is independent on the quantum entanglement. When  $r > 0$ , the fidelity of the clones is larger than  $1/2$ , thus the quantum  $1 \rightarrow M + (M-1)$  telecloning of coherent states is successful. The  $1 \rightarrow M + (M-1)$  coherent-state telecloning become reversible

and optimal with the fidelity  $M/(2M-1)$  for the clones and  $1/2$  for the anticlon when the EPR entangled state is perfect.

$N \rightarrow M + (M - N)$  *telecloning*. — We now address the most complicated case, the  $N \rightarrow M + (M - N)$  quantum telecloning, which produces  $M$  clones and  $M - N$  anticlones from  $N$  original replicas of a coherent state using  $2M$ -partite entanglement. The same multipartite entanglement Eq.3 is used for the quantum channels. The  $N$  replicas of a coherent state are stored the  $N$  modes  $\hat{a}_{in,1}, \dots, \hat{a}_{in,N}$ . In this scheme, we may consider to use a sender who holds the  $N$  input replicas and  $N$  entangled modes in the same set of the  $2M$ -partite reversible telecloning state, or  $N$  senders each of whom holds one of  $N$  input replicas and of the entangled modes in the same set. Each input replica is performed the joint measurement with a entangled mode. The sender(s) generate  $N$  amplitude- and phase-quadrature measurement results  $(x_1, p_1), \dots, (x_N, p_N)$  and inform other parties. After receiving these measurement results, each party first combines the measurement result  $x_s = \frac{\sqrt{2}}{N}(x_1 + \dots + x_N)$  and  $p_s = \frac{\sqrt{2}}{N}(p_1 + \dots + p_N)$ , and then displaces its entangled mode. The parties in the different set with the sender produce  $M$  clones with  $-g_x = g_p = 1$  and the parties in the same set with the sender produce  $M - N$  anticlones with  $g_x = g_p = -1$ . The fidelity of  $M$  clones and  $M - N$  anticlones is given by

$$\begin{aligned} \mathcal{F}_{clone}^{N \rightarrow M + (M - N)} &= \frac{NM}{NM + M - N + Ne^{-2r}}, \\ \mathcal{F}_{antic}^{N \rightarrow M + (M - N)} &= \frac{N}{N + 1}. \end{aligned} \quad (5)$$

The classical limit for  $N \rightarrow M + (M - N)$  quantum telecloning is  $\mathcal{F}_{class} = N/(N + 1)$ . The fidelity of the anticlones is  $\mathcal{F}_{antic} = N/(N + 1)$ , which is independent on the quantum entanglement. When  $r > 0$ , the fidelity of the clones is larger than  $N/(N + 1)$ , thus the quantum  $N \rightarrow M + (M - N)$  telecloning of coherent states is successful. The  $N \rightarrow M + (M - N)$  reversible telecloning requires the maximum EPR entanglement, which is optimal cloner with the fidelity  $MN/(MN + M - N)$  for the clones and  $N/(N + 1)$  for the anticlon[5].

*Asymmetric reversible telecloning*. — Let us now demonstrate how to make the reversible telecloning asymmetric. This is particularly interesting in the context of quantum cryptography where it enables Eve to choose a trade-off between the quality of her copy and the unavoidable noise that is added to the copy sent to the receiver. Here we only concentrate on  $1 \rightarrow 2 + 1$  asymmetric telecloning. The scheme of  $1 \rightarrow 2 + 1$  asymmetric telecloning is similar to symmetric telecloning as in Fig.1, in which only the vacuum noises  $\hat{v}_1$  and  $\hat{v}_2$  entering the beam splitters are replaced by another EPR entangled beams  $\hat{b}_{EPR1}$  and  $\hat{b}_{EPR2}$ . Bob and Claire produce the clones and Dan produced the anticlon, whose

fidelity is written as

$$\begin{aligned} \mathcal{F}_{clone}^B &= \frac{2}{2 + e^{-2r} + e^{-2r_b}}, \\ \mathcal{F}_{clone}^C &= \frac{2}{2 + e^{-2r} + e^{2r_b}}, \\ \mathcal{F}_{antic}^D &= \frac{2}{2 + e^{-2r_b} + e^{2r_b}}, \end{aligned} \quad (6)$$

where  $r_b$  is the squeezing factor of the EPR entangled beams  $\hat{b}_{EPR1}$  and  $\hat{b}_{EPR2}$ . It clearly shows that  $\mathcal{F}_{clone}^B > 2/3 > \mathcal{F}_{clone}^C$  and  $\mathcal{F}_{antic}^D < 1/2$  when  $r \rightarrow \infty$  corresponding to the reversible and optimal telecloning. This means that the more Bob achieves the information of the unknown state, the less Claire and Dan. The amount of information distributed in the remote receivers is controlled by the squeezing factor  $r_b$ . This confirms that the device indeed realizes the optimal asymmetric Gaussian telecloning of coherent states.

*Conclusion*. — We have introduced a scheme of CV reversible telecloning, which broadcast the information of an unknown state without loss from a sender to several spatially separated receivers exploiting multipartite entanglement as quantum channels. In this process, quantum information of an unknown state is distributed into  $M$  optimal clones and  $M - 1$  anticlones using  $2M$ -partite entanglement. This new scheme of implementing quantum state distribution nonlocally helps to deepen our understanding of the properties of quantum communication systems enhanced by the entanglement and its flexibility might have remarkable application in quantum communication and computation.

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